

Comment on “Einstein’s Other Gravity and the Acceleration of the Universe”

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We show that in the exponential $f(T)$ model studied by E. Linder [Phys. Rev. D **81**, 127301 (2010)], it is impossible to have the crossing of the phantom divide line $w_{\text{DE}} = -1$.

Recently, a new modified gravity theory, called as $f(T)$ with a torsion scalar T , has been proposed [1, 2] to explain the present accelerating expansion of the universe. The new theory is extended by the teleparallel gravity with the Weitzenböck connection instead of the Levi-Civita connection in general relativity. In this new theory, the gravity is no longer caused by curved spacetime but torsion and moreover, the field equations are only second order unlike the fourth order equations in the $f(R)$ theory. In Ref. [2], Linder has studied the exponential $f(T)$ model, given by

$$f \equiv f(T) = -\alpha T \left(1 - e^{pT_0/T}\right), \quad (1)$$

where $\alpha = [1 - \Omega_m^{(0)}]/[1 - (1 - 2p)e^p]$, $T_0 = T(z = 0)$ is the current torsion, and p is a constant with $p = 0$ corresponding to Λ CDM. In particular, he showed that the effective dark energy equation of state [2]

$$w_{\text{DE}} = -\frac{f/T - f_T + 2Tf_{TT}}{(1 + f_T + 2Tf_{TT})(f/T - 2f_T)} \quad (2)$$

crosses the phantom divide line $w_{\text{DE}} = -1$ for $p \leq 1$ from $w_{\text{DE}} > -1$ to $w_{\text{DE}} < -1$ in the opposite manner from the viable $f(R)$ models [3]. However, we find out that that the crossing of the phantom divide could not be realized in the exponential $f(T)$ model in Eq. (1). With the same set of the parameters as in Ref. [2], in Fig. 1 we show w_{DE}

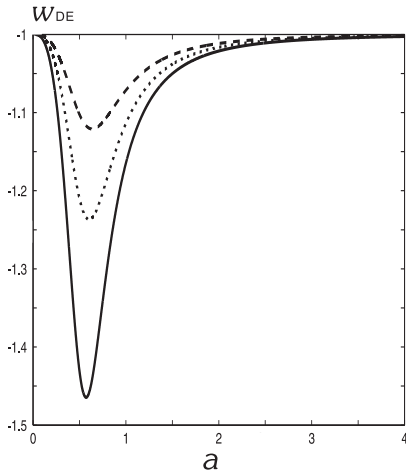


FIG. 1: Effective dark energy equation of state $w_{\text{DE}}(a)$ as a function of the scale factor a for the exponential $f(T)$ model, where the dash, dotted and solid lines represent $p = 0.25$, 0.5 and 1 , respectively.

in Eq. (1) as a function of the scale factor a for $p = 0.25$ (dash), 0.5 (dotted), and 1 (solid). It is clear that the equation of state is always less than -1 in contrast to the results of Fig. 1 in Ref. [2]. In fact, it can be shown [4] that the phase of the universe depends on the sign of the parameter p , i.e., for $p < 0$ the universe always stays in the non-phantom phase, whereas for $p > 0$ it always in the phantom phase. Our results also agree with those in Ref. [5].

We now discuss the reason why the crossing of the phantom divide cannot occur in the exponential $f(T)$ theory in Eq. (1) for $p \leq 1$. From Eq. (1), we find

$$f_T = \frac{df(T)}{dT} = -\alpha \left(1 - e^{pT_0/T} + \frac{pT_0}{T} e^{pT_0/T}\right), \quad (3)$$

$$f_{TT} = \frac{d^2f(T)}{dT^2} = \alpha \left(\frac{pT_0}{T}\right)^2 \frac{1}{T} e^{pT_0/T}. \quad (4)$$

To illustrate our results, we only concentrate on the limits of $0 < p \ll 1$ and $X \equiv pT_0/T \ll 1$. In this case, $T_0/T \lesssim 1$, which corresponds to the region from the far past to the near future. Consequently, Eqs. (1), (3) and (4) are approximately expressed as

$$\frac{f}{T} \approx \alpha \left(X + \frac{X^2}{2}\right), f_T \approx -\frac{\alpha X^2}{2}, T f_{TT} \approx \alpha X^2. \quad (5)$$

Substituting Eq. (5) into Eq. (2), we obtain

$$w_{\text{DE}} \approx -1 - \frac{3X^2/2}{X + 3X^2/2}, \quad (6)$$

where in deriving the approximate equality in Eq. (6) we have used $\alpha \sim O(1)$. From Eq. (6) we see that the universe always stays in the phantom phase (because $p > 0$ and $w_{\text{DE}} < -1$).

Finally, it is interesting to note that in the various $f(T)$ models extended from the viable $f(R)$ models in Ref. [6], all evolutions of w_{DE} show no crossing the phantom divide.

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- [1] G. R. Bengochea and R. Ferraro, Phys. Rev. D **79**, 124019 (2009) [arXiv:0812.1205 [astro-ph]].
- [2] E. V. Linder, Phys. Rev. D **81**, 127301 (2010) [arXiv:1005.3039 [astro-ph.CO]].
- [3] K. Bamba, C. Q. Geng and C. C. Lee, JCAP **1008**, 021 (2010) [arXiv:1005.4574 [astro-ph.CO]]; arXiv:1007.0482 [astro-ph.CO].
- [4] K. Bamba, C. Q. Geng and C. C. Lee, in preparation.
- [5] P. Wu and H. Yu, arXiv:1006.0674 [gr-qc].
- [6] R. J. Yang, arXiv:1007.3571 [gr-qc].